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LYCEN 2000-60, LYON
DFTT 28/2000, TORINO

Exchange-degenerate Regge trajectories: a fresh look from resonance and forward scattering regions

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Abstract

The exchange-degeneracy of the mesonic $f-, \omega-, \rho-$ and a_2- Regge trajectories dominant at moderate and high energies in hadron elastic scattering is analyzed from two viewpoints. The first concerns the masses of the resonances lying on these trajectories; the second deals with the total cross-sections of hadron and photon induced reactions. Neither set of data supports exact exchange-degeneracy.

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1 Introduction

A very convenient and useful method to group mesons and baryons in families with definite quantum numbers, makes use of the so called *Chew-Frautschi plot* (spin versus squared mass). It is a graphic representation of Regge trajectories for given quantum numbers. Early analyses of Regge trajectories hinted at remarkable properties [1]: they appear to be essentially linear and many of them coincide. The latter property came to be known as the principle of exchange-degeneracy (e-d) of Regge trajectories.

There are two kinds of exchange-degeneracy, qualified as *strong* and *weak*. In weak exchange-degeneracy, only the trajectories with different quantum numbers coincide. In strong exchange-degeneracy, in addition, the residues of the corresponding hadronic amplitudes coincide at the given pole in the $j-$ plane. It was soon realized that strong exchange-degeneracy may be violated (for theoretical arguments, see [1]) and indeed experimental confirmations of this violation occur.

Conclusive and definite statements about weak exchange-degeneracy, however, are not possible without sufficiently precise experimental information about the hadrons lying on each Regge trajectory. Therefore, lacking high precision data, general agreement with a weak exchange-degeneracy assumption (as well as with a linearity of meson Regge trajectories) was claimed in the past (see the references to old papers in [2]) and the hypothesis was applied repeatedly, for example, in models describing elastic scattering data (see references below). From this point of view, the most relevant trajectories are the $f-, \omega-, \rho-$, and the a_2-, which can variously be exchanged in the t-channel of many elastic reactions. These we are going to consider in what follows. The role of a unique Regge trajectory was repeatedly analyzed to describe hadron-hadron and photon-hadron total cross-sections in a most economical approach [3]. In spite, however, of an apparent agreement with the data, this model leads numerically to a quite large χ^2 when compared with more recent approaches [4, 5].

Today, the situation has changed somewhat. Three meson states are now known lying on each trajectory (except for the $\omega-$ trajectory for which we know only two states) and, moreover, some of their masses are measured with very high precision [2] even though the data on highest spin resonances have not yet been confirmed. We believe, however, that a fairly conclusive analysis can be performed using, on the one hand data in the resonance region (Section 2) and, on the other hand, data on (near forward) elastic scattering (Section 3).

Our conclusion (Section 4), will suggest that the combined analysis of all data supports a breaking of the weak exchange-degeneracy principle.

2 Resonance region

To examine the agreement of weak exchange-degeneracy with the available data in the resonance region, we first assume that the four trajectories $f-, \omega-, \rho-, a_2-$ are linear and coincide.

Writing the relevant exchange-degenerate linear trajectory as

$$\alpha_{e-d}(m^2) = \alpha_{e-d}(0) + \alpha'_{e-d}\ m^2\ , \tag{1}$$

we determine the intercept $\alpha_{e-d}(0)$ and the slope α'_{e-d} by fitting 11 resonances lying on $f-, \omega-, \rho-$ and a_2- trajectories. Using the MINUIT computer code, we find (the precision is estimated as the usual one-standard deviation error)

$$\alpha_{e-d}(0) = 0.4494 \pm 0.0007, \quad \alpha'_{e-d} = (0.9013 \pm 0.0011)\ \mathrm{GeV}^{-2}, \quad \text{with} \quad \chi^2/DoF = 117.9. \tag{2}$$

The data are taken from Ref. [2]. The very high value of χ^2/dof (dof stays for *Degree of Freedom* defined as the difference between the number of data points and the number of fitted parameters) is not surprising because *(i)* the data exhibit a known nonlinearity of the trajectories and *(ii)* the masses of the low lying resonances are measured with very high precision.

The trajectory one obtains is shown in Fig. 1 (solid line). For comparison, the trajectory with the parameters used in Ref. [6], $\alpha(m^2) = 0.48 + 0.88m^2$ (m in GeV), is also plotted (dashed line).

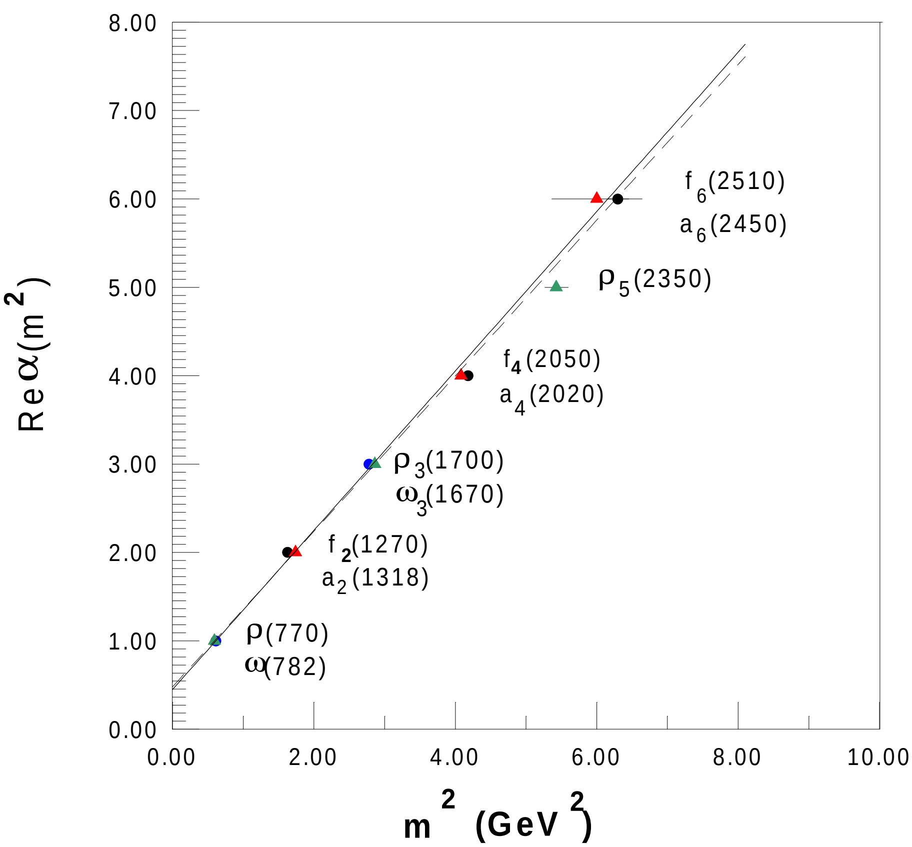

Figure 1: Chew-Frautschi plot for the fully exchange-degenerate $f - \omega - \rho - a_2$ trajectory. Solid line denotes the trajectory with the parameters obtained in our fit, dashed line is the trajectory from [6].

Our conclusion is, thus, that in spite of an apparent agreement with resonance data (plotted à la Chew-Frautschi), weak exchange degeneracy of the $f - \omega - \rho - a_2$ trajectories is not supported by the resonance data when a precise numerical analysis is performed.

In order to verify the possibility of a limited validity of exchange-degeneracy, we have considered a weaker version where the trajectories are grouped in pairs. Some of these combinations has been currently used to describe the behaviour of total cross-sections and of their differences. For convenience, we present the results in the Table 1 where all the 6 possible groupings in pairs are considered.

		ω	ρ	a_2
f	$\alpha(0)$	0.411	0.442	0.565
	$\alpha'(\mathrm{GeV}^{-2})$	0.963	0.944	0.835
	χ^2/dof	66.84	57.34	194.26
ω	$\alpha(0)$		0.445	0.456
	$\alpha'(\mathrm{GeV}^{-2})$		0.908	0.890
	χ^2/dof		84.14	13.48
ρ	$\alpha(0)$			0.482
	$\alpha'(\mathrm{GeV}^{-2})$			0.874
	χ^2/dof			1.30

Table 1. Intercepts $\alpha(0)$, slopes α' and χ^2/dof's obtained in the fits when exchange-degeneracy is assumed for each grouping in pairs of the trajectories. They are written at the intersection of the corresponding lines and rows.

For any grouping in pairs one can obtain the χ^2 from Table 1 because each pair is considered independently of the other. What would appear as a *natural* grouping introducing just two pairs of degenerate trajectories (one crossing even and one crossing odd $f - a_2 \equiv R_+$ and $\omega - \rho \equiv R_-$, as in [5, 7]), is clearly not supported by the resonance data under any reasonable *common* χ^2.

An obvious general conclusion follows from this very simple analysis: under a careful numerical investigation, there are no experimental evidences from the resonance region that the $f-, \omega-, \rho-$ and a_2- trajectories can be assumed to be *exchange-degenerate* (perhaps with the exception of the single pair $\rho - a_2$).

The available resonances are known with sufficiently good precision, allowing the determination of intercept and slope of each trajectory taken separately under the assumption of linearity [5]. We obtained the following parameters and the results are shown in Fig. 2.

$$
\begin{array}{lll}
\alpha_f(0) = 0.6971 \pm 0.041, & \alpha'_f = (0.8014 \pm 0.0018)\ \mathrm{GeV}^{-2}, & \chi^2/dof = 6.01, \\
\alpha_\omega(0) = 0.4359, & \alpha'_\omega = 0.9227\ \mathrm{GeV}^{-2}, & \text{(not fitted)}, \\
\alpha_\rho(0) = 0.4783 \pm 0.0011, & \alpha'_\rho = (0.8800 \pm 0.0017)\ \mathrm{GeV}^{-2}, & \chi^2/dof = 3.31, \\
\alpha_{a_2}(0) = 0.5116 \pm 0.0410, & \alpha'_{a_2} = (0.8567 \pm 0.023)\ \mathrm{GeV}^{-2}, & \chi^2/dof = 0.42.
\end{array}
\tag{3}
$$

[5]We should note that all trajectories, except the ω (for which only two resonances are known), deviate from of a linear behaviour (see also [9]). As a matter of fact, a deviation from linearity is dictated both by analyticity and unitarity and this has often been discussed in the past. For a recent discussion on the nonlinearity of the $f-$ trajectory and its influence on the intercept, see [10].

3 Forward scattering

3.1 Generalities

The exchange-degeneracy hypothesis for the $f - \omega - \rho - a_2$ trajectories can be checked also using elastic hadron scattering data. In particular, one can use forward scattering data, *i.e.* from the total cross-sections of hadron hadron-, of γ hadron- and of $\gamma\gamma$-collisions. Following the arguments given in [6] we restrict our analysis to the data on total cross-sections, $\sigma^{(t)}(s)$), excluding the ratios of the real to the imaginary parts of the forward amplitudes (due to its semi-theoretical determination).

Performing such an analysis requires an explicit parametrization for the amplitudes of the processes under investigation [6]. Like in the resonances-case, in order to check how well the exchange-degeneracy hypothesis works in a description of hadron and photon induced cross-sections, it is not sufficient to obtain agreement with the data which *looks good*. It is also necessary to compare this description with the one where the e-d assumption is removed. Clearly, removing the assumption of exchange degeneracy increase the number of parameters but the χ^2 referred to the number of degrees of freedom retains its comparative validity.

In addition to the main goal (to compare the e-d hypothesis with the data), we test the hypothesis of two models of Pomeron, each one with two components. One of them, explored in [6, 8], is *universal*, in the sense that its asymptotic component, growing with energy, contributes equivalently to all processes. The other one is *non-universal*: its two components contribute differently to each process, but with an universal ratio of these two components. We remark that the suggestion to consider models with a "two-component" Pomeron is not new. Many times, this idea was successfully applied (see [4, 11] and references therein).

Thus, we analyze the data using the following explicit expressions for the forward amplitudes $A_{ab}(s, t = 0)$ of the 12 elastic reactions

$$\begin{array}{rcl}
A_{p^\pm p}(s,0) &=& \mathcal{P}_{NN}(s) + f_{NN}(s) + a_{NN}(s) \mp \omega_{NN}(s) \mp \rho_{NN}(s),\\
A_{p^\pm n}(s,0) &=& \mathcal{P}_{NN}(s) + f_{NN}(s) - a_{NN}(s) \mp \omega_{NN}(s) \pm \rho_{NN}(s),\\
A_{\pi^\pm p}(s,0) &=& \mathcal{P}_{\pi N}(s) + f_{\pi N}(s) \mp \rho_{\pi N}(s),\\
A_{K^\pm p}(s,0) &=& \mathcal{P}_{KN}(s) + f_{KN}(s) + a_{KN}(s) \mp \omega_{KN}(s) \mp \rho_{KN}(s),\\
A_{K^\pm n}(s,0) &=& \mathcal{P}_{KN}(s) + f_{KN}(s) - a_{KN}(s) \mp \omega_{KN}(s) \pm \rho_{KN}(s),\\
A_{\gamma p}(s,0) &=& \delta[\mathcal{P}_{NN}(s) + f_{\gamma N}(s)],\\
A_{\gamma\gamma}(s,0) &=& \delta^2[\mathcal{P}_{NN}(s) + f_{\gamma\gamma N}(s)]
\end{array} \tag{4}$$

where

$$\mathcal{P}_{ab}(s) = i\{Z_{ab} + X\mathcal{P}(s)\} \qquad \text{for "universal" Pomeron,} \tag{5}$$

and

$$\mathcal{P}_{ab}(s) = iZ_{ab}\{\mathcal{P}(s) + X\} \qquad \text{for "nonuniversal" Pomeron.} \tag{6}$$

[6] The best way to analyze exchange-degeneracy would be to consider some linear combinations of $\sigma^{(t)}$ for several elastic processes. In principle, one can construct combinations that contain the contribution of one or two Reggeons and these could be compared with the experiment. The shortcoming of this procedure, however, lies in the fact that, usually, the required reactions are measured at different energies. As a consequence, while attractive, this procedure is heavily affected by the ambiguity of reconstructing data from interpolation. We shall not use this method.

We have considered two variants for the $s-$dependent Pomeron component, having in mind its properties in the complex angular momentum plane. The first one corresponds to a simple pole in the complex angular momentum plane with intercept $\alpha_{\mathcal{P}}(0) = 1 + \epsilon$, (the so-called *Supercritical Pomeron*)

$$\mathcal{P}(s) = (-is/s_0)^{\epsilon}, \qquad s_0 = 1\mathrm{GeV}^2. \tag{7}$$

The second variant corresponds to the *Dipole Pomeron*. In the j-plane it is described by a double pole with a unit intercept trajectory, $\alpha_{\mathcal{P}}(0) = 1$

$$\mathcal{P}(s) = \ln(-is/s_0). \tag{8}$$

For the secondary Reggeons we use the standard form

$$\mathcal{R}_{ab}(s) = \eta Y_{\mathcal{R}ab}(-is/s_0)^{\alpha_{\mathcal{R}}(0)-1} \quad R = f, a_2, \omega, \rho\ , \tag{9}$$

where $\eta = i$ for f and a_2 while $\eta = 1$ for ω and ρ.

The above amplitudes are normalized according to

$$\sigma_{ab}^{(t)}(s) = 8\pi \Im m A_{ab}(s, 0). \tag{10}$$

3.2 Results

We have taken into account the whole set of amplitudes and cross-section data for $p^{\pm}p, p^{\pm}n$, $K^{\pm}p, K^{\pm}n, \pi^{\pm}p, \gamma p$ and $\gamma\gamma$ interactions. In total there are 630 points at $\sqrt{s} \geq 5$ GeV, available in the Data Base of Particle Data Group [12]. No *wise selection* of any kind is attempted.

The values of the fitted parameters are given in Table 2 (for the universal Pomeron) and in Table 3 (for the non universal Pomeron). If exchange-degeneracy is assumed, all intercepts of $f-, a_2-, \omega-, \rho-$ Reggeons are equal (in Tables 2 and 3, we have labeled the common intercept as $\alpha_f(0)$). We do not give the curves for the total cross-sections because they are only illustrative and not very important for the case in point.

One can see that in all cases considered, non degenerate trajectories lead to a slightly better χ^2, even though for the universal Supercritical Pomeron the difference is small. It is interesting to note the very small value of $\epsilon = \alpha_{\mathcal{P}}(0) - 1$ for the model of non universal Pomeron with non degenerate trajectories (see column 8 in Table 2). As emphasized in [11, 4] the reason for this is that the Supercritical Pomeron approximates very closely Dipole Pomeron since

$$Z_{ab}[X + (-is/s_0)^{\epsilon}] \approx Z_{ab}[1 + X + \epsilon \ln(-is/s_0)],$$

where $Z_{ab}(1+X)$ and ϵZ_{ab} are very close to the corresponding parameters in the non universal Dipole Pomeron model with non degenerated trajectories (column 6 in Table 2).

	Universal Pomeron				Nonuniversal Pomeron			
	Dipole		Supercritical		Dipole		Supercritical	
	ne − d	e − d	ne − d	e − d	ne − d	e − d	ne − d	e − d
χ^2/dof	$.1080E+01$	$.1563E+01$	$.8180E+00$	$.8460E+00$	$.7832E+00$	$.1263E+01$	$.7867E+00$	$.9848E+00$
ϵ	$.0000E+00$	$.0000E+00$	$.1489E+00$	$.1688E+00$	$.0000E+00$	$.0000E+00$	$.1046E-02$	$.1157E+00$
Z_{NN}, GeV^{-2}	$-.2651E+01$	$.1953E+01$	$.2321E+01$	$.1056E+01$	$.6275E+00$	$.3240E+00$	$.6125E+03$	$.4131E+00$
$Z_{\pi N}$, GeV^{-2}	$-.3824E+01$	$.4375E+00$	$.8262E+00$	$.4625E+00$	$.4211E+00$	$.2006E+00$	$.4131E+03$	$.2549E+00$
Z_{KN}, GeV^{-2}	$-.3952E+01$	$.1251E+00$	$.5141E+00$	$.3357E+00$	$.3944E+00$	$.1752E+00$	$.3887E+03$	$.2217E+00$
X, GeV^{-2}	$.6221E+00$	$.2803E+00$	$.6020E+00$	$.1652E+00$	$-.3554E+01$	$.5092E+01$	$-.1004E+01$	$.1583E+01$
$\alpha_f(0)$	$.8063E+00$	$.5575E+00$	$.5331E+00$	$.4842E+00$	$.7729E+00$	$.5327E+00$	$.7808E+00$	$.4856E+00$
Y_{fNN}, GeV^{-2}	$.1021E+02$	$.9498E+01$	$.8297E+01$	$.3619E+01$	$.1076E+02$	$.1204E+02$	$.1091E+02$	$.4397E+01$
$Y_{f\pi N}$, GeV^{-2}	$.8618E+01$	$.6801E+01$	$.4920E+01$	$.2052E+01$	$.6010E+01$	$.5518E+01$	$.6154E+01$	$.1765E+01$
Y_{fKN}, GeV^{-2}	$.7748E+01$	$.5290E+01$	$.3172E+01$	$.1215E+01$	$.4673E+01$	$.3028E+01$	$.4842E+01$	$.6628E+00$
$Y_{f\gamma N}$, GeV^{-2}	$.2796E-01$	$.1783E-01$	$.1177E-01$	$.4685E-02$	$.2857E-01$	$.2324E-01$	$.2937E-01$	$.6710E-02$
$Y_{f\gamma\gamma}$, GeV^{-2}	$.7411E-04$	$.2599E-04$	$.2175E-05$	≈ 0	$.7186E-04$	$.3439E-04$	$.7535E-04$	$.3598E-05$
α_ω	$.4899E+00$	$=\alpha_f(0)$	$.4546E+00$	$=\alpha_f(0)$	$.4471E+00$	$=\alpha_f(0)$	$.4484E+00$	$=\alpha_f(0)$
$Y_{\omega NN}$, GeV^{-2}	$.3790E+01$	$.3190E+01$	$.4275E+01$	$.1527E+01$	$.4384E+01$	$.3457E+01$	$.4364E+01$	$.1537E+01$
$Y_{\omega KN}$, GeV^{-2}	$.1283E+01$	$.1066E+01$	$.1413E+01$	$.5018E+00$	$.1439E+01$	$.1123E+01$	$.1433E+01$	$.4947E+00$
$\alpha_\rho(0)$	$.5961E+00$	$=\alpha_f(0)$	$.5971E+00$	$=\alpha_f(0)$	$.5810E+00$	$=\alpha_f(0)$	$.5800E+00$	$=\alpha_f(0)$
$Y_{\rho NN}$, GeV^{-2}	$.1647E+00$	$.2195E+00$	$.1745E+00$	$.9267E-01$	$.1977E+00$	$.2749E+00$	$.1972E+00$	$.1104E+00$
$Y_{\rho\pi N}$, GeV^{-2}	$.5945E+00$	$.6503E+00$	$.5918E+00$	$.2906E+00$	$.5964E+00$	$.6625E+00$	$.5974E+00$	$.2824E+00$
$Y_{\rho KN}$, GeV^{-2}	$.2972E+00$	$.3360E+00$	$.2888E+00$	$.1523E+00$	$.2984E+00$	$.3420E+00$	$.2996E+00$	$.1470E+00$
$\alpha_{a_2}(0)$	$.7336E+00$	$=\alpha_f(0)$	$.6166E+00$	$=\alpha_f(0)$	$.5926E+00$	$=\alpha_f(0)$	$.6053E+00$	$=\alpha_f(0)$
Y_{a_2NN}, GeV^{-2}	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	≈ 0
Y_{a_2KN}, GeV^{-2}	$.5572E-01$	$.1492E+00$	$.1068E+00$	$.8771E-01$	$.1199E+00$	$.1665E+00$	$.1115E+00$	$.8469E-01$
δ	$.3427E-02$	$.3048E-02$	$.3060E-02$	$.3028E-02$	$.3404E-02$	$.3070E-02$	$.3425E-02$	$.3052E-02$

Table 2. Results of fits to forward elastic data using the universal and non universal Pomeron comparing the results in the case of exchange-degeneracy (e-d) with those when exchange-degeneracy is not assumed (ne-d).

4 Conclusion

Our conclusions are very brief. From the available data on the mesons lying on the $f-, \omega-$, $\rho-$ and a_2- trajectories, we conclude that the exchange degeneracy of these trajectories (assumed linear) is not compatible with the data.

Concerning the forward scattering data, the situation is less clear cut because a comparatively good description of the data can be obtained under both hypotheses. Qualitatively, we can say that different constant terms Z_{ab} in Eqs.(4),(5) correct for the universal behavior of a unique Regge term. Even so, the fits with non degenerate trajectories lead to somewhat better χ^2's which can be taken as an indication in favor of non degenerate Regge trajectories. For a more definite conclusion we need more precise data on meson-nucleon and proton (K-meson)-neutron cross-sections at higher energies. Even more conclusive, perhaps, would be to compare fits with and without exchange degeneracy involving *all* data both at $t = 0$ and at $t \neq 0$. This analysis, performed in [13] puts much more stringent constraints on the free parameters as was often emphasized by the present authors; all best fits favor non exchange degeneracy solutions.

Thus, given that any model for scattering amplitudes should be in agreement with both types of data, from spectroscopy and from total cross-sections we conclude that the hypothesis of *exact exchange degeneracy* , even in its weak formulation, is not supported by the present data. In spite of this, due to its great economy in the number of parameters, exchange degeneracy retains its usefulness in practical calculations when only a rough approximation is sufficient.

Acknowledgments E.M. would like to thank the Université Claude Bernard and IPNL for the kind hospitality and financial support provided to him during this work.

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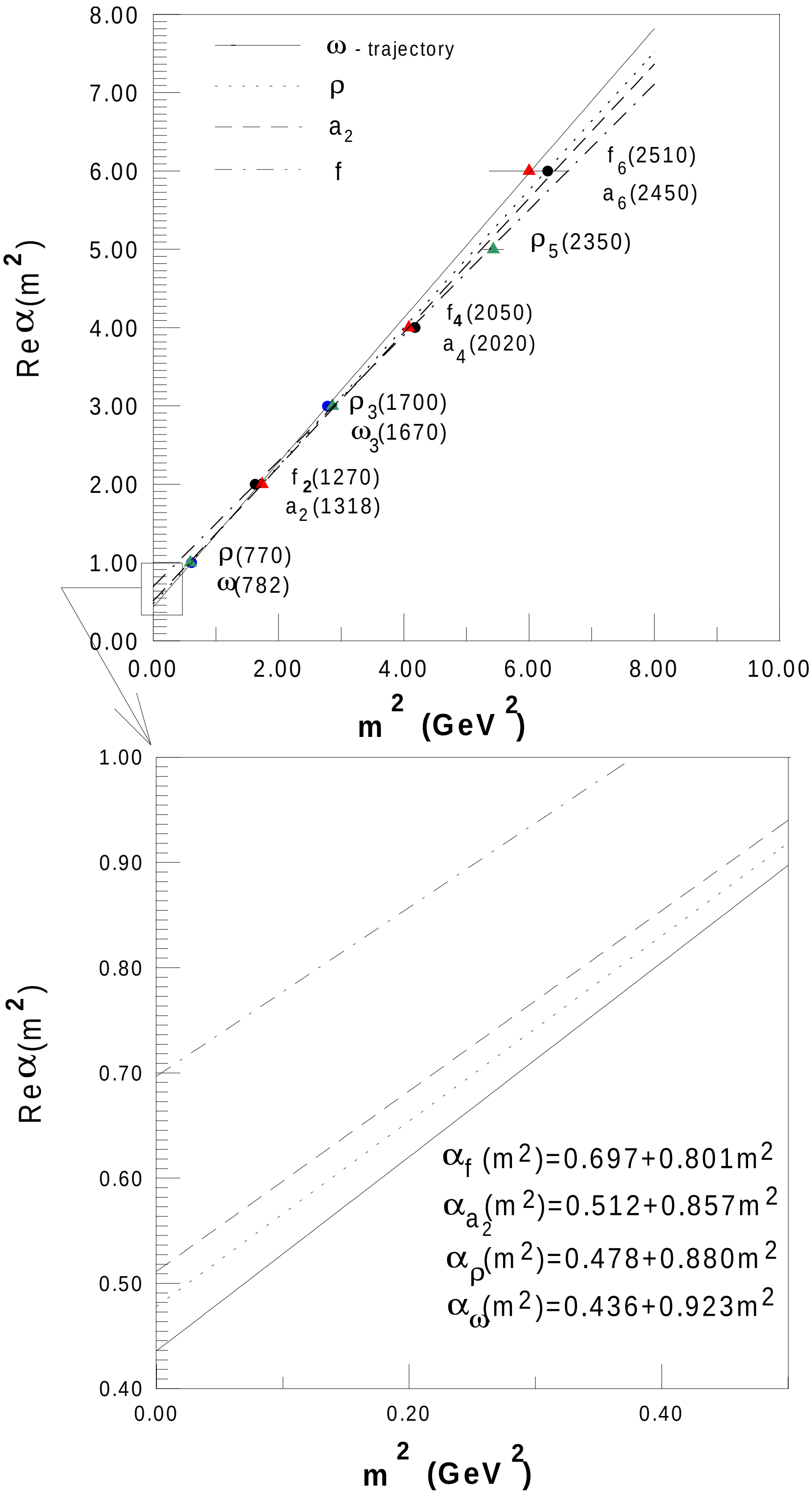

Figure 2: Chew-Frautschi plots for f-, ω-, ρ- and a_2 Regge trajectories taken separately assuming linearity (the figure below is an enlargement for low m).